

Maximal Ergodic Theorems for Operators with Finite Peripheral Spectrum

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Abstract

Maximal and variational inequalities play a fundamental role in ergodic theory, harmonic analysis, and the study of operator semigroups. In this talk, we present new maximal ergodic inequalities for positive contractions on noncommutative L_p -spaces associated with semifinite von Neumann algebras. Assuming that the numerical range of the underlying operator is contained in a closed polygon with vertices on the unit circle, we establish noncommutative maximal inequalities extending the celebrated results of Junge–Xu and their subsequent generalization by Bekjan. In the commutative setting, our results recover and significantly extend Stein’s classical maximal ergodic theorem.

We also discuss a q -variational inequality for contractively regular operators satisfying a suitable resolvent estimate with finite peripheral spectrum. This extends earlier work of Le Merdy and Xu and provides stronger quantitative information on the convergence of ergodic averages. Finally, we present a noncommutative weak-type maximal inequality for convolution powers, extending a classical theorem of Calderón and Bellow to the noncommutative setting. The proofs combine new polynomial identities, square function estimates, and a refinement of Stein’s analytic family method.