

A MACKEY-TYPE DECOMPOSITION FOR WALLED BRAUER ALGEBRAS AND APPLICATIONS TO SCHUR ALGEBRAS

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ABSTRACT. Let A be an algebra over a field K and B, C be two subalgebras of A . A Mackey-type decomposition of a B -module M over the algebra A is a decomposition of the C -module $\text{Res}_C^A \text{Ind}_B^A(M)$ into direct sum of smaller C -modules. We establish a Mackey-type decomposition for modules over the walled Brauer algebras, $B_{r,t}(\delta)$, $r, t \in \mathbb{N}$, $\delta \in K^*$. The walled Brauer algebras were first introduced in 1989 by Koike and Turaev independently. Motivated by the classical Mackey decomposition theorem for group algebras of finite groups, our work extends the diagram algebra framework developed by Hartmann for Brauer algebras. Using the cellularly stratified structure of $B_{r,t}(\delta)$, we define appropriate induction and restriction functors and derive a decomposition theorem for permutation modules. As an application, we obtain a decomposition formula for homomorphism spaces between permutation modules over $B_{r,t}(\delta)$. In our ongoing work, using the decomposition of homomorphism spaces between permutation modules over the walled Brauer algebra, we are trying to give a dimension formula for the Schur algebra of the walled Brauer algebra and give a basis of the Schur algebra with some nice combinatorial properties.

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