Ph. D. Thesis Defence

On Well-posedness and Control Problem for Some Phase-Field Models

By

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Abstract

Many natural and industrial process involves different phase living together like oil and water in a glass, cream mixing into coffee. Phase-field models are class of diffuse interface models used to describe system with coexisting phases. This talk presents the main analytical results of my doctoral work on diffuse interface models for the dynamics of two incompressible, immiscible viscous fluids. The interface between the fluids is modeled as a region of finite thick-ness, leading to Cahn–Hilliard type phase-field formulations coupled with fluid flow equations. Depending on the underlying medium, I study the Cahn–Hilliard–Navier–Stokes system and its damped variant. First, I establish well-posedness, regularity, and continuous dependence of solutions for the Cahn–Hilliard–Navier–Stokes system with nonhomogeneous boundary condi-tions. These results form the basis for the analysis of an optimal boundary control problem, for which the existence of optimal controls and first-order necessary optimality conditions are derived. Next, I address local controllability of a one-dimensional Cahn-Hilliard-Burgers system using localized interior controls, relying on Carleman estimates and fixed point ar-guments. Finally, I study a three-dimensional damped Cahn-Hilliard-Navier-Stokes system, proving existence, energy equality, and uniqueness of weak solutions under both regular and singular potentials.

References

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