

Completeness of grid translates of functions in Orlicz spaces

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Abstract

We consider the Orlicz space $L^\Phi(\mathbb{R}^n)$ defined using a N -Young function Φ which satisfies the Δ_2 -regular condition. Let $\{\Lambda_k\}_{k \in \mathbb{N}}$ be a sequence of discrete finite subsets of $\mathbb{R}^n$ such that $\Lambda_k \subset \Lambda_{k+1}$ and $\bigcup_{k \in \mathbb{N}} \Lambda_k$ is dense in $\mathbb{R}^n$. Let $f \in C_0(\mathbb{R}^n) \cap L^\Phi(\mathbb{R}^n)$ be a real valued function. Suppose that there exists $x \in \mathbb{R}^n$ for which there exists a sequence of neighborhoods of $f(x)$ whose inverse images shrink regularly to x . Then we show that $\text{span}[\bigcup_{k \in \mathbb{N}} \{f^k(\cdot - \lambda) : \lambda \in \Lambda_k\}]$ is dense in $L^\Phi(\mathbb{R}^n)$. Furthermore, this result is extended to non-Euclidean cases such as Homogeneous groups, Riemannian symmetric spaces of non-compact type of finite real rank, and Damek-Ricci spaces.

References

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