Finite Element Analysis and Approximation of the Fluid–Structure Interaction Problem

Nishant Ranwan

PHD211027

Abstract

This talk is focused on analyzing the nonlinear coupled fluid-structure interaction (FSI) problem motivated by its application in the blood flow through the coronary artery. An FSI problem is a multi-physics problem that consists of fluid dynamics and structure mechanics. The incompressible Navier–Stokes equations describe the fluid part of the FSI problem. The equations describe the fluid flow as a homogeneous, unsteady, incompressible Newtonian fluid in terms of velocity and pressure fields. The elastodynamic equations describe the structural part using the linear St. Venant–Kirchhoff model.

We analyze the spatially semi-discrete finite element approximation of the FSI problem. We assume that the fluid and solid subdomains are stationary in order to perform finite element analysis. During spatial discretization, the grad-div stabilization term is included, and the divergence form of the convective term is used for the fluid part. We prove the existence of the finite element solution for the coupled problem, derive stability estimates, and a priori finite element error estimate. We prove that the derived error bound does not depend on the Reynolds number. Following this, we analyze the continuous interior penalty stabilization (CIP) technique for the FSI problem. We present the pressure-robust and convection-robust error bound in the spatially semi-discrete case. To derive such a robust error bound, a divergence-free finite element pair is used for the spatial discretization of the fluid problem, with stabilization terms incorporated during discretization. The stabilization term also includes the tangential jump of the convective term across the fluid-solid interface, involving both fluid and solid velocities. We derive an error bound for which the order of convergence is $k \geq 2$ for the fluid and solid velocities in the energy norm, where k is the polynomial degree of finite element functions.

In the fully-discrete case, we study a loosely coupled Robin-Robin partitioned scheme. The coupled problem is split into fluid and structure sub-problems based on Robin-Robin boundary conditions at the interface. We derive the stability bound for the finite element solutions and perform the a priori finite element error analysis in the fully discrete case. The derived error bound is of an $\mathcal{O}(h^k+\sqrt{\Delta t})$ convergence, i.e., the rate of convergence is sub-optimal. We perform numerical simulations of a two-dimensional FSI benchmark problem and demonstrate pressure-wave propagation. The fluid and structure sub-problems are solved in a fully decoupled strategy. Numerical experiments are conducted to examine the spatial and temporal convergence behavior of the partitioned scheme under consideration.