Maximum principles for matrix-valued regular functions of a quaternionic variable.

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Abstract

A quaternionic matrix-valued regular function is a map $F : \Omega \to M_n(\mathbb{H})$ whose entries are (left) regular functions of a quaternion variable, where Ω is a domain in $\mathbb{H}$. This talk aims to bring out some maximum norm principles for such functions. We derive an SVD type decomposition theorem for such functions, using the notion of maximizing vectors. Further we prove a Fisher type approximation theorem for regular functions $f : \mathbb{B} \to \overline{\mathbb{B}}$ that are continuous on $\partial\mathbb{B}$, in terms of convex combinations of finite Blaschke products over $\mathbb{H}$ ($\mathbb{B}$ being the quaternionic unit ball). This in turn yields a Fisher type approximation theorem for an $n \times n$ matrix-valued regular function on the quaternionic unit ball, where each entry of the matrix satisfies the same condition as above.

References

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