

Spaces of invariant and ergodic measures

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Abstract

The main aim of this exposition is to introduce and study the properties of two of the basic spaces of measures in the ergodic theory of maps. We first study the space of all Borel probability measures (denoted by $M(X)$) on a given compact space X . Then we proceed to studying some special elements of the space $M(X)$ under the dynamics of a continuous map $T : X \rightarrow X$. The two spaces under consideration in this regard are $M(X, T)$, the space of all T -invariant measures and $E(X, T) \subset M(X, T)$, the space of all measures with respect to which T is ergodic. After introducing these notions, we explore some of the topological properties of the above spaces, study some examples and culminate by stating a result which gives a geometric characterisation of members of $E(X, T)$. The talk shall be accessible to masters and PhD students alike. Familiarity with some basics in measure theory and functional analysis shall help.

References

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- [Wal82] Peter Walters. *An introduction to ergodic theory*. Vol. 79. Graduate Texts in Mathematics. Springer-Verlag, New York-Berlin, 1982, pp. ix+250. ISBN: 0-387-90599-5.