Interpolation but lifting, commutants and operators

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Abstract

All the interpolation discussed are concerned with disk or polydisc or unit ball.

So well in this talk I wanna introduce something which I have encountered earlier this year, while many people are familiar with Lagrange interpolation on real variable, the same proof generalizes to the complex plane, and the Lagrange polynomials being the analytic, and well so the next question which arose and was introduced and solved by Pick is that given a set of data, can we interpolate it using analytic function but with a small catch being that now we want our analytic function to be on the unit ball of $\mathbb{H}^\infty(D)$. This leads to the failure of Lagrange polynomials satisfying it in general, makes us search elsewhere, even though this problem was solved by Pick (1916), another proof for a much more general tool for a class of interpolation problems which well extend beyond pick interpolation was provided using operator theory tools by Sarason (Generalized interpolation, 1967) with the use of commutant lifting approach which use some basics of dilation theory and is closely related to motivations from the Hardy space, specifically the Hardy Hilbert space of the unit disk $\mathbb{H}^2(D)$.

Well one might wonder what's the niceness of this approach if the problem could be solved and characterized completely using classical Hardy Hilbert space and matrix techniques, well the issue turns out to be when our interpolation is over polydisc or unit ball of $\mathbb{C}^n$ instead of $\mathbb{C}$ there were issues with providing characterizations, a larger complete characterization was not provided until last year by the work of Jaydeb Sarkar, Bhattachaterjee, Deepak K D whose motivations arose from Sarason's method, though some charcterizations were provided earlier. And secondly, the solution by Sarason is very interesting from an operator theoretic point of view and solves and generalizes a larger class of interpolation in this talk we will explore thew results and the idea of the proof.